Improved Low-Altitude Constellation Design Methods

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This paper provides methods for designing low-altitude constellations that offer good coverage properties with fewer satellites than any other known constellation. Specifically, it addresses global, double below-the-horizon coverage and global, single above-the-horizon coverage with heavy coverage at high latitudes. The methods entail optimally meshing coverage circles near the equator in order to minimize the number of satellites required to provide the desired coverage. As a result, one can analytically determine good constellations. This offers an improvement over the often-used trial-and-error method of optimization. The methods described also provide for simple calculations of minimum coverage levels.

Introduction

LOW-ALTITUDE, high-inclination satellite constellations are currently receiving much attention. They are being considered most notably for strategic defense. Because of time, distance, and visibility constraints, most space-based weapon and sensor constellations must be in low-altitude, high-inclination orbits. Because satellites are expensive to build and deploy, it is beneficial to minimize the number required to perform any specific mission. Often, this translates into the problem of determining the minimum number of satellites needed to satisfy specific coverage criteria.

The basic objective of the research described in this paper is to determine constellations that provide continuous global coverage with the minimum number of satellites. Specifically, the contribution here is for single above-the-horizon (ATH) coverage with heavy coverage at high latitudes and on double below-the-horizon (BTH) coverage.

Walker "delta" constellations, named for their inventor,[1] are the standard for designing constellations for these purposes. For this reason, coverage provided by these new methods will be compared to coverage provided by Walker delta constellations. Although Walker constellations originally were developed to provide global coverage, the same configurations of satellites usually are referred to as Walker constellations even if they do not provide global coverage.

A Walker delta constellation has all of its satellites in circular orbits at the same altitude and the same inclination, with a certain number of orbit planes. Each plane has the same number of satellites spaced evenly in the plane. Furthermore, all planes are spaced evenly around the equator, and the relative phasing between satellites in adjacent planes is the same for all planes. Within these constraints, one may vary the inclination, relative phasing, and number of planes to minimize the required number of satellites for a specific application.

Others have considered constellations that differ from Walker delta constellations. Some constellations have ascending nodes of all orbit planes between 0 and 180 deg instead of between 0 and 360 deg, as in Walker delta constellations. Walker also considered such constellations[2] but found the delta patterns to be more advantageous for his purposes. Nevertheless, different designs may offer advantages in other cases. Adams and Rider[3] developed constellations that reduce the number of satellites required to attain global, single BTH coverage,[4] but do not improve on the number required for double BTH coverage over the Walker delta.

Walker delta constellations may not necessarily be advantageous for all missions, in that they do not always maximize the coverage provided by a given number of satellites. Although the symmetric positioning of satellites at a particular time seems to suggest optimal coverage, there is no guarantee that this symmetry does, in fact, optimize the positioning of subsatellite points on the sphere one is trying to cover. In addition, at any time during which no satellite lies directly above the equator, symmetry of positions is destroyed. Highly inclined Walker delta constellations have another disadvantage. Satellites in nearby planes crossing the equator move in nearly opposite directions. When they approach the equator, their coverage circles may have a small area of overlap and a large area of coverage. However, a short time later, when the satellite coverage circles cross over each other, a hole in coverage will appear unless there is a third satellite close by. Consequently, Walker delta constellations may provide alternating periods of good and poor coverage as the satellites orbit. If sensors provide only ATH coverage, this problem is compounded by crossings of the nadir holes under each satellite. Some early studies of alternate ATH constellations are found in Refs. 5 and 6. They apply only in special cases.

Walker delta constellations do yield the minimum known number of satellites required for some coverage criteria, such as double BTH coverage. However, even knowing that the best constellation is a Walker delta constellation leaves the problem of determining which Walker delta constellation is best. For a given number of satellites at a fixed altitude, one may vary the inclination, number of orbit planes, and relative phasing between satellites in adjacent planes. The trial-and-error method, which is often used, is not very efficient. Reference 7 derives some analytic means to determine good Walker delta constellations. However, these methods do not necessarily yield the best Walker delta constellation.

New Design Methods

The essential feature of the constellation design methods described here is that satellites' coverage circles mesh optimally near the equator, and that the coverage increases at higher latitudes because satellites travel in nearly the same direction. For satellites in highly inclined orbits, the region near the equator is the most difficult region to cover since that is where adjacent orbit planes are farthest apart. Satellites with coverage circles that mesh in a common orbiting plane or in nearby planes provide "streets of coverage," which are lanes of continuous coverage (Fig. 1). This idea was first introduced by R. D. Lüders[8] for BTH coverage.

Adjacent planes are separated by as much as possible without leaving coverage gaps between adjacent streets of

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Presented as Paper 87-0498 at the AAS/AIAA Astrodynamics Specialist Conference, Kalispell, MT, Aug. 10–13, 1987; received July 10, 1987; revision received March 15, 1988.

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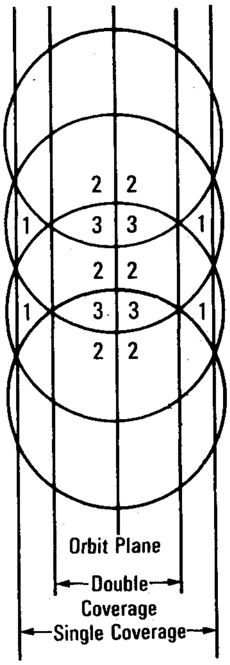
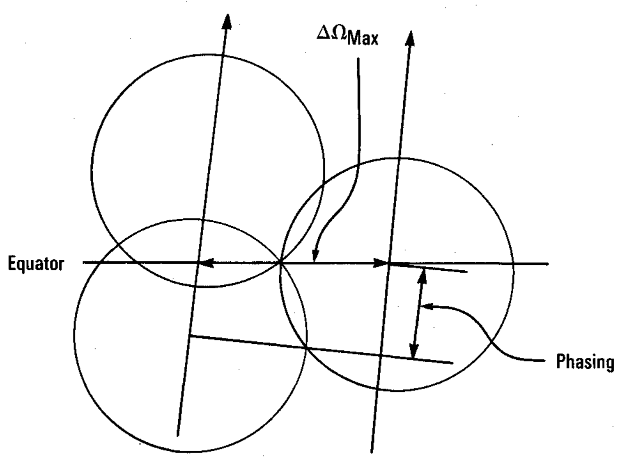
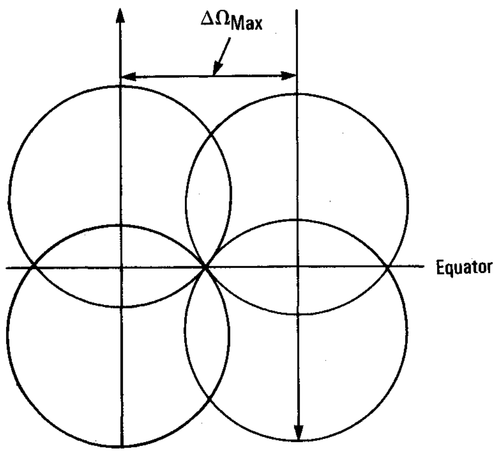


Fig. 1 Streets of coverage.



a) Planes moving in the same direction



b) Planes moving in the opposite direction

Fig. 2 Meshing of coverage between adjacent planes. Satellites in each plane provide a street of coverage.

coverage (Fig. 2a). In any region where satellites in adjacent planes are moving in opposite directions, the planes must be closer together (Fig. 2b). Adams and Rider[3] used similar meshing approaches to determine analytically the number of satellites required to provide global, single BTH coverage. This paper extends the arguments in order to study global ATH single-coverage constellations that use nonsymmetric placing of satellites and global BTH double-coverage Walker delta constellations. All constellations considered here consist of satellites in circular orbits, all at the same altitude and inclination.

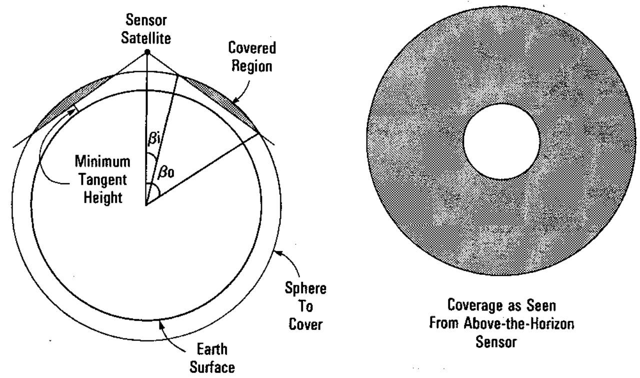


Fig. 3 Above-the-horizon coverage.

Single ATH Coverage

Overview

Under the label of ATH coverage, this paper considers coverage provided by sensor satellites that are constrained in that they cannot see in any direction where the Earth is the background (Fig. 3). The closest distance to the Earth's limb that they can view an object is referred to as the minimum tangent height. Since coverage increases as altitudes of targets increase, coverage can be analyzed by considering the coverage provided on an imaginary sphere centered at the center of the Earth. If the surface of this sphere is covered, so is everything above it.

If each satellite can see to a minimum tangent height of b above the surface of the Earth, then the inner and outer radii of the coverage circles (β_i and β_o, respectively) are given by (in Earth-central angles)

$$\beta_o = \cos^{-1}[(R_e + b)/r] - \cos^{-1}[(R_e + b)/R_{\text{eff}}] \quad (1)$$

$$\beta_i = \cos^{-1}[(R_e + b)/r] + \cos^{-1}[(R_e + b)/R_{\text{eff}}] \quad (2)$$

where R_{eff} is the radius of the sphere to be covered, R_e the radius of the Earth, and r the radius of the sensor satellite constellation. The covered region is the annulus between the two radii.

Figure 4 shows three meshing possibilities for ATH coverage. The constellations resulting from the methods described here are not Walker delta constellations. The purpose of the mesh is to cover nadir holes with nearby satellites and to cause these satellites to move as groups in order to assure continuous coverage. Each meshing determines the maximum allowable spacings between satellites in the same orbiting plane and maximum allowable spacings between adjacent planes. In turn, these determine the number of satellites required to attain single global coverage. All orbit planes have ascending nodes between 0 and 180 deg, so that ascending satellites have coverage annuli that mesh to provide coverage on one side of the sphere, while descending satellites provide coverage on the other side of the sphere.

Because all orbit planes have ascending nodes between 0 and 180 deg, polar constellations are typically the best since nonpolar constellations may leave coverage gaps (Fig. 5). If there are only two orbit planes, however, any inclination may be used, provided the angle between the orbit planes remains the same. One may turn each diagram in Fig. 4 by 90 deg to get groups of orbit planes rather than groups of satellites in a

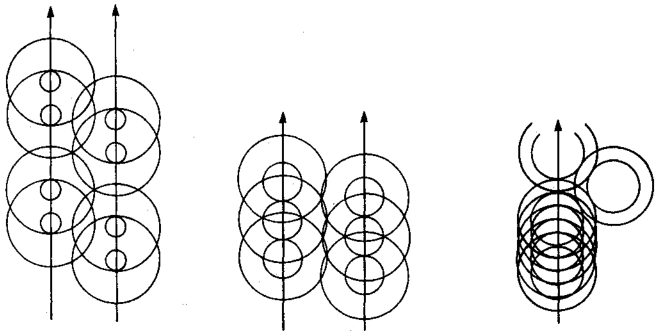


Fig. 4 Designing constellations for continuous above-the-horizon coverage.

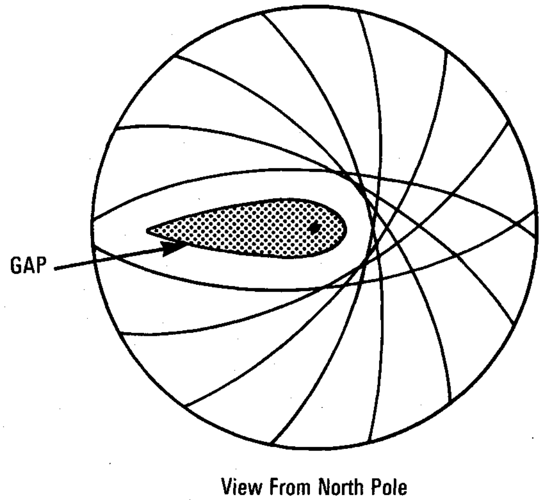


Fig. 5 Loss of coverage at high northern latitudes for nonpolar orbits.

plane. However, this would cause coverage holes to overlap satellites move away from the equator.

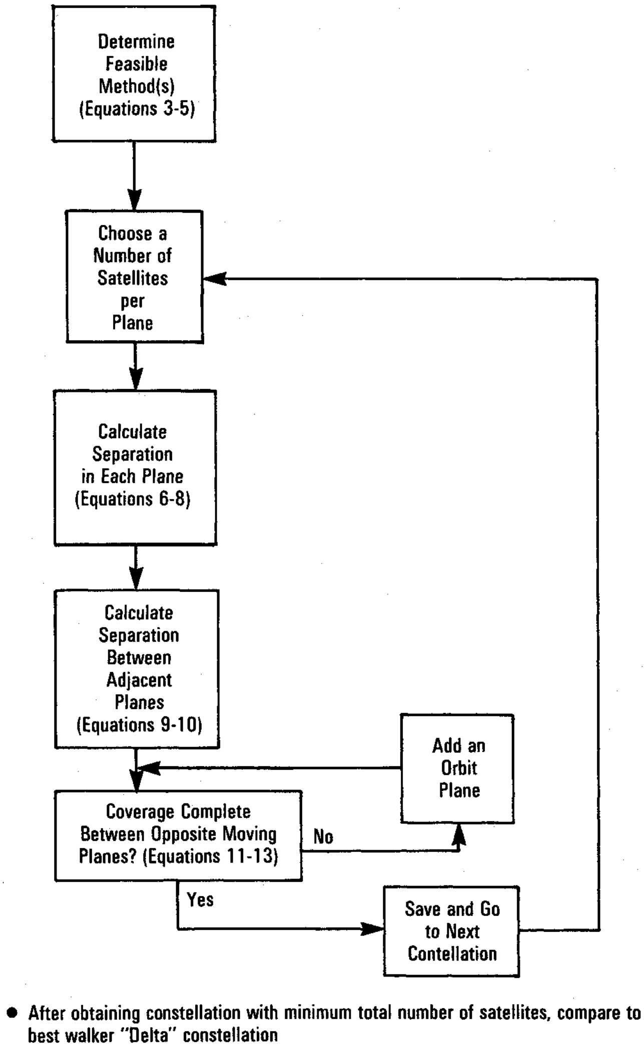


Fig. 6 Single above-the-horizon coverage constellation design method.

Design Methods

Figure 6 shows a flowchart of the method used to design these ATH constellations. The following description details the method. First, the conditions that allow for each method are described. In the first method—henceforth referred to as the "pair method"—nadir holes are small enough so that pairs of satellites cover each other's hole. This is feasible if, and only if,

$$\beta_o \geq 3\beta_i \tag{3}$$

A certain number of pairs are spread out in an orbit plane. With the second method—henceforth referred to as the "even method"—two adjacent satellites cover the hole of a given satellite, and all satellites are evenly spaced in an orbit plane. This method was described for only one orbit plane in Ref. 6. This paper extends the even method to global coverage. Two conditions are necessary for this method to be feasible. They are

$$\cos^{-1}\left[\frac{\cos\beta_o}{\cos\beta_i}\right] \geq 2\beta_i \tag{4}$$

and the requirement that an integer number of satellites per plane (I) exists, which satisfies

$$2\beta_i \leq 2\pi/I \leq \cos^{-1}\left[\frac{\cos\beta_o}{\cos\beta_i}\right] \tag{5}$$

The third method—henceforth referred to as the "packet method"—involves placing satellites in packets that cover one another's nadir holes. The packet method always works and is especially useful when the other methods are not feasible. When more than one method can work, one should analyze each to find the one that minimizes the total number of satellites.

Given one of the three meshing methods as shown in Fig. 4, the separation of satellites in a plane is formulated. With the pair method, the separation between two satellites of a pair is $\beta_o - \beta_i$. The separation between pairs can be calculated for a given number of satellites in a plane. If the number of pairs is n, then this separation is given by

$$\alpha = [2\pi - n(\beta_o - \beta_i)]/n \tag{6}$$

Because satellites form pairs, there must be an even number of satellites in each orbit plane unless one satellite is added to the plane to decrease the interpair separation α.

For the even method, the satellite separation is given by

$$\alpha = 2\pi/I \tag{7}$$

where I is the number of satellites in an orbit plane.

For the packet method, the separation between satellites within a packet is given by $\beta_o - \beta_i$, as in the pair method. Now, however, several satellites are necessary to cover all the nadir holes in a packet. Packets may be separated by a larger distance, up to

$$\alpha = 2\pi/n - (\beta_o - \beta_i)(j-1) \tag{8}$$

where n is the number of packets and j the number of satellites in a packet. There is a minimum number of satellites per packet needed to cover the nadir holes. However, one may be able to lower the total number of satellites by adding an additional satellite to one or more packets and increasing the separation between adjacent orbit planes.

Simple spherical trigonometric equations determine the separation between orbit planes and the relative phasing between satellites in adjacent planes. The separation at the equator between orbit planes having satellites that move in the same direction is

$$\Delta\Omega_{\perp} = \cos^{-1}[\cos\beta_o/\cos(\alpha/2)] + \beta_o \tag{9}$$

The plane separation when satellites are moving in opposite directions is

$$\Delta\Omega_{\text{opp}} = 2\cos^{-1}[\cos\beta_o/\cos(\alpha/2)] \tag{10}$$

These separations are similar to those identified in Ref. 9.

Because of the phasing of satellites in an orbiting plane, the most difficult point to cover between satellites moving in opposite directions may not be at the equator. In this case, Eq. (10) does not give the maximum allowable plane separation. The actual allowable plane separation (Ω') is found from the latitude (ϕ) of "worst coverage." Let

$$p = \cos^{-1}[\cos\beta_o/\cos(\alpha/2)] \tag{11}$$

Each orbit plane covers an angle

$$\gamma = \sin^{-1}[\sin p/\sin(\pi/2 - \phi)] \tag{12}$$

so that

$$\Omega' = 2\sin^{-1}(\sin p/\cos\phi) \tag{13}$$

The relative phasing is chosen to cause a satellite in an adjacent plane to be midway between the large gaps in the original plane. Any phasing that causes all large gaps in the original plane to be covered by satellites in the next plane is satisfactory, and the relative phasing need not be the same for all planes. This is useful when one wishes to maximize Ω' in Eq. (13) for cases where some phasings do not work.

There are two possible relative phasings between two adjacent planes when the pair method is used. If one satellite is added to a pair, the number of possible phasings between two planes increases to four. There is only one possible relative phasing between two planes for the even method. For the nominal packet method, the number of possible relative phasings between two adjacent planes is equal to the number of satellites in a packet. If satellites are added to packets, the number of possible phasings is twice the number per nominal packet. If the added number of satellites is enough to add a satellite to each packet, the number of satellites per nominal packet is increased by one and the results are otherwise equivalent.

Results

In order to determine the usefulness of the new design methods, 30 cases were analyzed (Table 1). A computer program was written to derive the best constellation using the new design. It runs on a PC in less than 30 s, a substantial improvement over the usual trial-and-error method. The new methods improve on the Walker designs, in some cases, for providing global coverage with the minimum number of satellites. More importantly, they generally provide better high-latitude coverage and allow analytical constellation design.

The Walker delta constellations shown in Table 2 are the best in that they give global coverage and provide better high-latitude coverage than any other Walker delta constellation that gives global coverage and uses the same number of satellites as the new method.

Table 1 Above-the-horizon constellations for global coverage

Case	Altitude, km	Minimum target altitude, km	Minimum tangent height, km	No. of satellites required: Walker "delta"	No. of satellites required: New design
1	500	100	60	82	80
2	500	200	60	37	36
3	500	300	60	26	30
4	500	200	100	48	48
5	500	300	100	31	33
6	1,000	100	60	56	60
7	1,000	200	60	27	30
8	1,000	300	60	19	21
9	1,000	200	100	34	36
10	1,000	300	100	20	27
11	1,500	100	60	54	54
12	1,500	200	60	21	24
13	1,500	300	60	15	16
14	1,500	200	100	33	36
15	1,500	300	100	16	16
16	2,000	100	60	42	48
17	2,000	200	60	21	24
18	2,000	300	60	14	18
19	2,000	200	100	24	28
20	2,000	300	100	17	18
21	2,500	100	60	39	42
22	2,500	200	60	20	20
23	2,500	300	60	14	18
24	2,500	200	100	26	24
25	2,500	300	100	17	18
26	3,000	100	60	37	36
27	3,000	200	60	19	18
28	3,000	300	60	15	16
29	3,000	200	100	23	22
30	3,000	300	100	16	16

Table 2 Orbit parameters for Walker "delta" constellations (maximizing high-latitude coverage and giving continuous global coverage: same number of satellites as in new design method)

Case	No. of satellites, N	Inclination, deg	No. of orbit planes	Walker phasing No., F^a
1	80	65	80	1
2	36	90	3	1
3	30	60	15	12
4	48	65	48	44
5	33	85	3	0
6	60	60	30	27
7	30	65	3	2
8	21	60	21	9
9	36	55	18	33
10	27	65	3	0
11	54	50	27	24
12	24	55	24	22
13	16	75	16	14
14	36	65	18	15
15	16	45	2	5
16	48	60	48	46
17	24	50	24	22
18	18	50	18	17
19	26	45	26	24
20	18	45	18	16
21	42	45	42	40
22	20	40	20	18
23	18	50	18	16
24	24	45	24	23
25	16	45	16	14
26	36	40	36	34
27	18	35	18	16
28	16	40	16	14
29	22	35	22	20
30	16	35	16	14

[a]Relative phasing between planes is $F*360/N$ deg.

The constellations that use the new method and provide global coverage with the minimum number of satellites are detailed in Table 3. All the new constellations are polar. In cases where there are two orbit planes, other inclinations can give equivalent global coverage but will decrease the polar coverage.

The new constellations use all three methods described in this paper. Some require extra satellites in a plane to minimize the total number of satellites. Finally, cases 12, 16, 24, 26, 27, and 29 in Table 3 had some relative phasings that work and some relative phasings that do not work. The chosen phasing does not satisfy Eq. (10) at the equator but does satisfy Eq. (13) at the latitude where the worst meshing occurs.

Table 4 compares coverage by latitude between the Walker delta designs and the new designs. If one defines "best" coverage as a requirement to obtain single coverage everywhere, with double coverage from the pole down to the lowest latitude possible (and then triple coverage, if possible, etc.), the new ATH constellation design methods generally improve on the Walker delta design. The new methods also allow quick analytical constellation design and provide an estimate of the number of satellites needed for Walker designs.

Double BTH Coverage

Overview

Below-the-horizon coverage implies that satellites are not restricted to areas above the Earth's limb but rather can access any areas that meet the specified coverage criteria (range, elevation angle, etc.). One may attempt to develop constellations that use design methods similar to those for the ATH constellations in this paper, but Walker delta constellations generally provide double coverage with a smaller number of satellites. This is because satellites with ascending nodes from 0 to 180 deg can mesh well to provide one level of

Table 3 Above-the-horizon global coverage constellations

Case	No. of planes	Spacing between ascending nodes, deg	Satellites per plane	Method	Packets per plane, satellites per packet	Extra satellites added to plane	Relative phasing between planes, deg
1	4	48.326	20	Packet	5,4	0	55.111
2	3	61.560	12	Even	N/A	N/A	15.000
3	3	61.931	10	Pair	N/A	0	51.407
4	4	46.824	12	Pair	N/A	0	40.003
5	3	61.973	11	Pair	N/A	1	19.131
6	3	61.739	20	Packet	4,5	0	70.482
7	3	61.169	10	Even	N/A	N/A	18.000
8	3	67.503	7	Pair	N/A	1	39.457
9	3	62.724	12	Packet	4,3	0	65.006
10	3	61.386	9	Even	N/A	N/A	20.000
11	3	63.207	18	Packet	3,6	0	91.852
12	2	90.014	12	Packet	4,3	0	45.000
13	2	91.998	8	Even	N/A	N/A	22.500
14	3	63.286	12	Packet	3,4	0	90.010
15	2	93.200	8	Even	N/A	N/A	22.500
16	2	90.224	24	Packet	3,7	3	15.407
17	2	92.316	12	Packet	3,4	0	95.526
18	2	93.012	9	Packet	3,3	0	90.814
19	2	94.184	14	Packet	3,4	2	23.322
20	2	98.831	9	Packet	3,3	0	88.115
21	2	95.956	21	Packet	3,7	0	98.222
22	2	94.769	10	Packet	2,4	2	42.632
23	2	92.584	9	Packet	3,3	0	90.814
24	2	91.626	12	Packet	2,5	2	59.990
25	2	96.446	9	Packet	2,4	1	40.798
26	2	92.256	18	Packet	2,8	2	51.778
27	2	96.337	9	Packet	2,4	1	72.237
28	2	96.263	8	Packet	2,4	0	136.222
29	2	94.663	11	Packet	2,5	1	64.992
30	2	102.935	8	Packet	2,4	0	132.173

Table 4 Minimum coverage level by latitude (same number of satellites for Walker and new)

	Latitude, deg																			
	Walker										New									
Case	0	10	20	30	40	50	60	70	80	90	0	10	20	30	40	50	60	70	80	90
1	1	1	0	0	2	2	1	2	3	4	1	1	1	1	1	1	2	4	4	4
2	0	0	1	1	1	1	3	3	3	4	1	1	1	1	1	1	3	3	4	3
3	1	1	1	1	2	2	2	2	2	3	1	1	1	1	1	2	3	3	4	4
4	1	1	1	1	2	2	2	2	3	4	1	1	1	1	1	1	2	4	4	5
5	1	1	1	1	1	2	3	4	4	3	1	1	1	1	1	1	3	3	4	4
6	1	1	1	1	1	1	1	1	2	6	1	1	1	1	1	1	3	3	3	3
7	1	1	1	2	2	2	2	2	3	5	1	1	1	1	1	2	3	4	3	3
8	1	1	1	1	1	1	1	1	3	3	1	1	1	1	1	1	2	3	3	3
9	1	1	1	1	1	1	1	2	2	3	1	1	1	1	1	2	3	3	3	3
10	1	1	1	2	2	2	2	3	4	3	1	1	1	1	1	2	3	3	3	3
11	1	1	1	1	1	1	1	2	2	3	1	1	1	1	1	2	3	3	3	3
12	2	1	1	1	1	1	2	2	3	4	1	1	1	1	1	2	2	2	2	3
13	1	1	1	1	1	1	2	1	2	2	1	1	1	1	1	2	2	2	2	2
14	1	1	1	2	1	1	1	2	4	6	1	1	1	1	1	2	3	3	3	3
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2
16	1	1	1	1	1	2	2	2	5	4	1	1	1	1	1	2	2	2	2	2
17	2	2	2	2	2	2	2	2	3	4	1	1	1	1	1	2	2	2	2	2
18	2	2	2	2	2	2	2	2	3	4	1	1	1	1	1	2	2	2	2	2
19	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	2	2	2	2	2
20	2	2	2	1	1	1	2	2	2	2	1	1	1	1	1	2	2	2	2	2
21	2	2	2	2	2	2	2	2	2	4	1	1	1	1	1	2	2	2	2	2
22	2	2	1	1	1	1	2	2	1	2	1	1	1	1	1	2	2	2	2	2
23	2	2	2	2	2	2	2	2	4	4	1	1	1	1	2	3	2	2	2	2
24	1	1	1	1	1	0	1	2	3	4	1	1	1	1	1	1	2	2	2	2
25	1	1	1	1	0	1	0	2	2	2	1	1	1	1	1	1	2	2	2	2
26	2	1	1	1	0	1	1	2	2	4	1	1	1	1	1	1	2	2	2	2
27	1	1	1	1	1	0	1	1	1	2	1	1	1	1	1	1	2	2	2	2
28	2	2	2	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2
29	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	1	2	2	2	2
30	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	2	2	2	2	2

Table 5 Double-coverage Walker constellations

No. of satellites per plane	Minimum No. of satellites required	No. of planes	Inclination, deg	Example relative phasing, deg
19	133	7	90	Any
18	126	7	90	Any
17	153	9	75	0.0
16	144	9	70	17.5
15	135	9	70	0.0
14	126	9	70	2.857
13	130	10	70	0.0
12	120	10	75	6.0
11	110	10	70	6.545
10	110	11	70	13.091
9	117	13	70	12.308
8	128	16	70	14.063
7	117	17	70	18.151
6	108	18	70	23.333
5	115	23	70	31.304
4	108	27	70	53.333
3	108	36	70	83.333
2	112	56	70	28.929
1	105	105	70	161.143

coverage, while satellites with ascending nodes from 180 to 360 deg provide the second layer of coverage. In this case, there is no need to move adjacent orbit planes near 0 and 180 deg closer together, as was done previously for ATH coverage.

Reference 7 derived analytical means to determine good Walker delta constellations. However, the number of satellites required was not minimized. Analysis of the meshing properties of Walker delta constellations provides a means of determining which of them is optimal. Five cases, each of which is based on the number of satellites in each orbiting plane, need to be considered:

1) Satellites in each orbiting plane provide a street of double coverage.

2) Satellites in each orbiting plane provide a street of single coverage.

3) There are too few satellites in a plane to provide a street of single coverage, but the separation of coverage circles is less than 2β, where β is the radius of the coverage circles.

4) Separation of coverage circles is greater than 2β.

5) There is only one satellite per orbiting plane.

By considering optimal equatorial meshing of satellites for each of these cases, one can choose a number of satellites per plane, determine the required number of planes, and, thus, determine how many satellites are required. Comparing the number of satellites for each of the five cases determines the overall minimum number required for global BTH double coverage. Thus, analysis of satellite meshing enables one to reduce the candidates for the best Walker delta constellations. Two additional simplifications also result. First, analyzing meshing gives exact numbers for the number of satellites required. Second, analysis of meshing yields the optimal inclination of satellites. No such information is given a priori by Walker delta constellation constraints.

Design Methods

The radius of each coverage circle (Earth-central angle) is given by

$$\beta = \cos^{-1}[(R_{\text{eff}}^2 + r^2 - R^2)/(2R_{\text{eff}}\, r)] \tag{14}$$

where R_{eff} is the radius of the sphere to be covered (Earth radius or above), r the radius to the satellite, and R the satellite range. For the case where minimum elevation angle of the satellite (as seen from the surface of the sphere to be

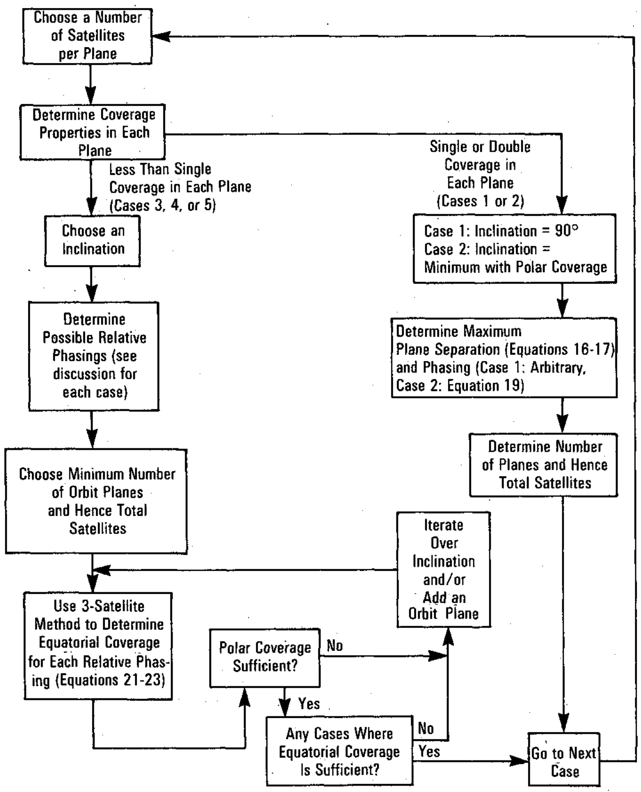


Fig. 7 Double below-the-horizon coverage constellation design method.

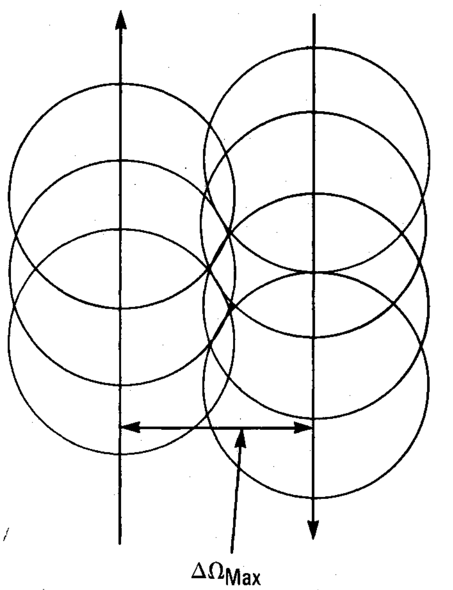


Fig. 8 Continuous double-coverage rings.

covered) determines the coverage, the correct equation is

$$\beta = \pi/2 - \sigma - \sin^{-1}[(R_{\text{eff}}/r)\,\sin(\pi/2 + \sigma)] \qquad (15)$$

where σ is the minimum elevation angle.

Figure 7 shows a flow diagram for the method used to derive the best Walker delta constellation for double BTH coverage. The following describes the method in more detail. First consider case 1. Suppose each plane contains enough satellites to provide a street of double coverage. This is pictured in Fig. 8. Note that inclinations near 90 deg work best since other inclinations will cause holes in coverage near the equator, thus requiring additional orbit planes. Also, since orbits are polar, the constellation must consist of an odd number of planes so that their longitudes of ascending node

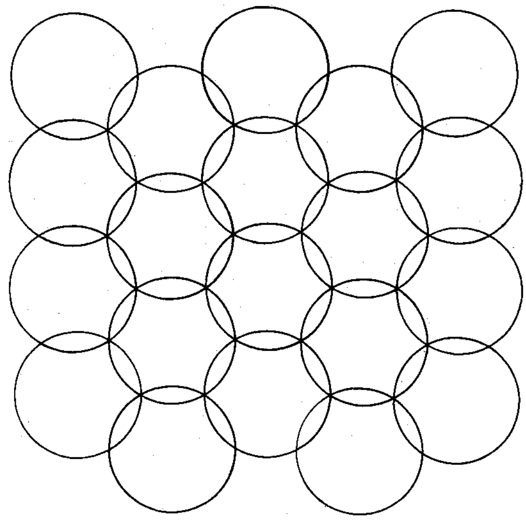


Fig. 9 Ideal circle meshing.

cause the coverage lanes to mesh properly. Both of these effects are due to the fact that orbit planes are spaced evenly around the equator from 0 to 360 deg. The maximum allowable separation at the equator between an orbit plane going north and an adjacent plane going south is given by

$$\Delta\Omega = \cos^{-1}[\cos\beta/\cos(\alpha/2)] + \cos^{-1}[\cos\beta/\cos\alpha] \qquad (16)$$

where α is the separation between satellites in a plane. The relative phasing does not affect the global double coverage, so that any phasing works as well as another. Equation (16) enables one to determine the number of planes required to provide global double coverage.

Next, suppose there are only enough satellites in each plane to provide a street of single coverage (case 2). Satellites moving north across the equator have coverage circles that mesh to provide the first level of coverage and satellites moving south overlap with a second layer of coverage. Separation at the equator is given by

$$\Delta\Omega_{\max} = 2\sin^{-1}[\sin(\Delta\Omega_{\perp}/2)/\sin i] \qquad (17)$$

where

$$\Delta\Omega_{\perp} = \cos^{-1}[\cos\beta/\cos(\alpha/2)] + \beta \qquad (18)$$

and i is the orbit inclination.

The relative phasing between planes (θ) is given by

$$\theta = \alpha/2 - 2\cos^{-1}[\cos(\Delta\Omega_{\max}/2)/\cos(\Delta\Omega_{\perp}/2)] \qquad (19)$$

where α is the angular spacing between satellites in a plane. Note that, because of the Walker constraint, discontinuities between planes at 0 and 180 deg no longer exist since single-coverage layers are required from 0 to 360 deg. Also note that, in a Walker constellation, θ must be chosen so that it is the same for all orbit planes. In general, this is inconsistent with Eq. (19). Thus, Eq. (19) serves only as an approximation for θ. Each street of single coverage overlaps with a street of single coverage provided by satellites whose ascending node differs by close to 180 deg. These two overlapping streets result in double coverage. One also must be sure that the inclination is not lower than that which guarantees continuous polar and near-polar coverage.

Suppose now that there are not enough satellites in each plane to give a street of continuous single coverage, but that the satellite separation is less than 2β (case 3). Optimal meshing of circles can still result in global double coverage. Again, satellites moving north on one side of the Earth provide the first level of coverage, while satellites moving

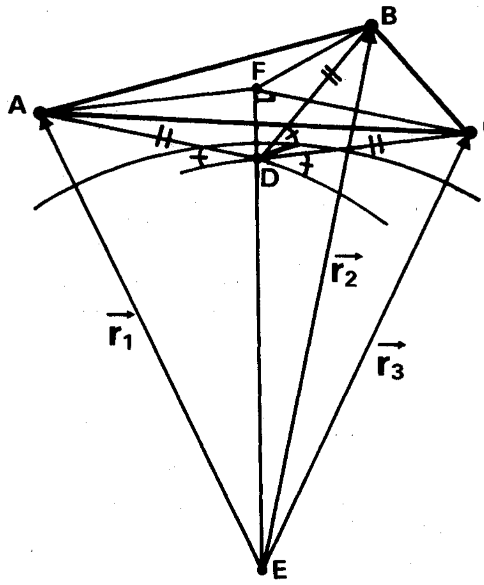


Fig. 10 Determining coverage of the worst viewing location.

south on the same side provide the second level. When meshing, only circles near the equator need be considered. Because all the circles are closer together at higher latitudes, coverage will necessarily improve there. Thus, one may analyze meshing by studying a planar optimum meshing as is shown in Fig. 9. The case where each ring provides continuous coverage is seen by viewing from the bottom to the top of the page. Rotating the figure yields a coverage pattern with fewer satellites per plane. In case 3,

$$\Delta\Omega_{\perp} = \cos^{-1}[\cos\beta/\cos(\alpha/2-\beta)] \tag{20}$$

When the inclination is 90 deg, relative phasing is $\alpha/2$. Nonpolar constellations may provide improved coverage [Eq. (17)]. However, since the meshing requires the intersection of coverage circles of satellites in more than two orbiting planes, choosing an optimum phasing between planes 1 and 3 precludes optimal placing of satellites in plane 2. Hence, separations must be altered to compensate. One may test nonpolar Walker delta constellations, adjusting the phasing as before, to see if any of these are better. Since the Walker constraints generally preclude optimal meshing, approximations to the optimal phasing will be used. In order to determine which constellation is best, one must verify which of the candidates provides double coverage. The method for doing this is now described.

In order to verify that a Walker delta constellation provides global coverage, place a satellite at 0° latitude, 0° longitude. Two satellites nearby and to the east determine a spherical triangle on the sphere to be covered. Once the three initial positions are known, one can easily obtain the relative positions of the three satellites as a function of time. A short iteration over mean anomaly finds the positions that cause the worst coverage from these three satellites. The worst viewing location occurs at a point where all three satellites are at the same elevation in the sky, if one chooses the correct three initial points. One may then find the angular distance θ_{max} between this worst viewing location and the three satellite position vectors using the following equations:

$$\boldsymbol{r}_i = \cos\lambda\,\cos\phi\hat{\imath} + \sin\lambda\,\cos\phi\hat{\jmath} + \sin\phi\hat{k} \tag{21}$$

where λ is the longitude, ϕ the latitude, and $\boldsymbol{r}_i$ the Earth-centered ith position vector. θ_{max} is given by

$$\theta_{max} = \cos^{-1}\{\pm\boldsymbol{r}_i \cdot [(\boldsymbol{r}_2 - \boldsymbol{r}_1)\times(\boldsymbol{r}_3 - \boldsymbol{r}_1)]\} \tag{22}$$

This equation follows from the fact that the three triangles formed by the Earth center, the satellite positions, and the ground point are identical (Fig. 10). So are the three triangles joining satellite positions to the ground point to the location where the line from the Earth's center through the ground point intersects the plane containing the three satellites. θ_{max} is equivalent to the radius of a Walker circumcircle.[1] Note that θ_{max} is the same, regardless of which $\boldsymbol{r}_i$ is used. One chooses the sign so that

$$\cos\theta_{max} > 0 \tag{23}$$

Walker constraints guarantee that θ_{max} determines the worst coverage provided by the constellation. A value for θ_{max} that is within the coverage range of the satellite guarantees continuous double coverage everywhere provided that the poles are doubly covered. Again, this is because a single layer of coverage is moving north across the equator while the second layer is moving south.

Now suppose there is more than enough room between coverage circles in each plane to fit another coverage circle (case 4). A meshing similar to those in Fig. 9 still may be obtained. One may fit two, three, or more coverage circles between two that are in the same orbiting plane. One of these satellites has a coverage circle closest to the second satellite in the original plane. This one must have phasing placing it nearly adjacent to the second satellite in the original plane. For example, if there are two satellites between satellites in a plane, the third satellite moving upward is nearly adjacent to the second satellite in the original plane. In this case then, the Walker phasing must be such that, three planes to the right, there is a satellite with nearly the mean anomaly of the second satellite in the first plane. As another example, if there are three satellites between coverage circles in the same plane, the fourth satellite going up will be nearly adjacent to the second satellite in the first plane. Only a few Walker phasings fit these constraints for a given number of satellites per plane. Equations (21–23) may then be used to determine the coverage levels.

The only situation still to be considered is that with only one satellite per plane (case 5). One may view the total number of planes as consisting of a certain number of sets of planes. Each set consists of satellites whose ascending nodes increase by the smallest increment and continues until one reaches a satellite at near-zero phasing relative to the first satellite. This satellite is the first satellite of the second set. The satellites in the first set must form a street of coverage. The approximate number of sets of satellites and numbers of satellites in each set may be found by calculating the number of coverage circles required to cover a great circle. For example, a coverage circle of 40 deg in diameter means that one cannot have fewer than nine satellites per set or fewer than nine sets without having holes. The number of satellites per set must be an integer, but the number of sets may be a fraction.

After one full set of satellites, the mean anomaly returns to nearly zero. This ensures that the first satellite of the second set will be nearly adjacent to the first satellite of the first set. The cases are reduced further by the requirement that there be no significant overlap of coverage circles within a set.

By trying different combinations of numbers of satellites in a set and total numbers of sets, one can find the candidate constellations. One may then use Eqs. (21–23) to determine the coverage levels, keeping in mind that one must verify polar coverage for any nonpolar constellation.

Results

In order to illustrate these arguments, some results are given for BTH double-coverage constellations. One case in particular is detailed. Suppose the satellites are at a 1500 km altitude, with a 3000 km range, and covering the Earth. A constellation designed using adaptations of ATH meshing techniques with ascending nodes from 0 to 180 deg would require 120 satellites. However, the best Walker delta constellation gives continuous double coverage with only 105 satellites. The best Walker delta constellations as a function of the number of satellites per plane are in Table 5. The methods described in this paper enable one to reduce the number of possible phasings for one satellite per plane from 105 to 18 and to analytically obtain most of the results. It is interesting to note that, in most cases, the lower inclination of 70 deg improves coverage over higher inclinations. Seventy degrees is close to the lowest inclination possible before the constellation loses polar coverage. Thus, the beneficial effect of an inclination below 90 deg [Eq. (17)] outweighs the harmful effect of requiring several orbit planes for coverage of one area (which requires that one move the planes closer together). In many cases for a given number of satellites per plane, many phasings work as soon as one reaches the number of satellites where the optimal phasing works.

The total number of satellites required for global double coverage depends heavily on the number of satellites per plane. For example, eight satellites per plane do poorly because they do not provide a street of coverage but instead leave small holes between satellite coverage circles. Coverage improves as one moves away from eight satellites per plane. Finally, it is interesting to note that one satellite per plane yields the lowest total number of satellites. This is reasonable, since there are 105 possible phasings, which increases the likelihood that some will yield nearly the ideal meshing of Fig. 9. On the other hand, if there are, say, four satellites per plane, they are constrained to be 90 deg apart, regardless of whether or not that angle is close to one of optimal meshing.

Conclusions

There are a number of areas that would make interesting future study. In the above-the-horizon (ATH) coverage case, one could study why certain Walker delta constellations have coverage annuli that mesh well to provide good global coverage. In the below-the-horizon (BTH) double-coverage case, further study of the coverage-circle meshing of various Walker delta constellations should make constellation design even simpler. One might also study other types of coverage, such as double ATH coverage and coverage of certain latitude zones.

This paper presents methods for designing good global, ATH continuous single-coverage constellations and for designing good global, BTH continuous double coverage constellations. The objective was to develop methods for designing the best constellations in a quick, analytical fashion, thereby avoiding the usual trial-and-error method.

Three ATH methods are introduced: the pair method, the even method, and the packet method. Conditions for use of each method are presented, together with complete instructions for their use in designing constellations. Thirty examples are presented comparing ATH constellations derived using the new methods to those resulting from Walker delta designs. The ATH methods usually yield better high-latitude coverage while maintaining continuous global coverage than the best Walker delta constellation with the same number of satellites. The new methods allow easy determination of good constellations without running computer simulations.

For many-satellite, BTH double-coverage constellations, the Walker delta design yields the smallest known requirement for total numbers of satellites. This paper presents methods for determining the best Walker delta designs via equations and geometries without large computer simulations to determine the coverage levels.

Acknowledgments

This document reports research sponsored in part by the Strategic Defense Initiative Organization (SDIO) under Contract MDA 903-85-C-0049. The views, opinions, and findings contained in this paper are those of the authors and should not be construed as those of SDIO or of ANSER. The authors are grateful to the reviewers for their many constructive comments related to this paper. The paper is much improved as a result. The authors also wish to thank Ms. Joyce Hottle for her large contribution.

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